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Production of femtosecond single-bunched electrons by laser wakefield acceleration

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Abstract

Design of a femtosecond single-bunched electron linac is described based on laser wakefield acceleration (LWA) for study of solid-state physics and chemistry. In this acceleration scheme, a short laser pulse drives plasma waves. The potential of the waves then accelerates charged particles. The bunch length of the accelerated particles is proportional to the plasma wavelength. The present design using a 100 fs–2 TW T^3 laser gives a 30 MeV–10 pC single bunch with an rms bunch length of less than 10 fs. It accelerates and compresses a high-current beam from an rf photocathode. Use of a chicane and a plasma klystron is proposed for the compression. Some practical problems concerning the design are also presented. © 1997 Elsevier Science B.V.

1. Introduction

High-energy particles accelerated by a laser wakefield have recently been observed. Experiments with high-density plasmas based on self-modulated laser wakefield acceleration (LWA) have attained an acceleration of 45 MeV over a 0.3 mm dephasing length, or 150 GeV/m acceleration gradient [1,2]. Another acceleration scheme based on self-guiding of laser pulses in plasmas has attained higher gain (> 100 MeV) with more modest acceleration gradient [3]. However, these schemes are not yet applicable to high-energy linear colliders, because other accelerator parameters, such as current, emittance, etc., still remain critical issues.

The LWA, however, has some other features, in addition to a high acceleration gradient and compact size. The bunch length can be shorter than 10% of the plasma wavelength, or around 10 fs in the standard LWA and around 1 fs in the self-modulated LWA. On the other hand, there are some demands for short-bunched linacs. An example exists in the field of chemistry in order to study the initial phase of radiation-induced reactions [4]. This technique is called pulse radiolysis, in which sample materials are bombarded by short particle bunches, and the resultant species are analyzed spectroscopically by short-pulsed laser beams. It is said that phenomena such as ionization and excitation occur within a femtosecond after bombardment, and that the thermalization of electrons and the relaxation of molecular vibration occur in the range between sub-femtosecond and a nanosecond. Radical–electron pairs or geminate pairs produced by bombardment recombine and become excited states, also within a picosecond. Short-bunched beams are essential for studying these transient phenomena.

Another application of the LWA is generation of femtosecond X-ray pulses either by Thomson scattering [5] or by plasma undulators [6]. Such X-ray pulses combined with sampling techniques enable dynamic studies of phenomena in solids in the femtosecond range using X-ray diffraction techniques [7].

Such an application as the pulse radiolysis requires not only particle beams, but also laser beams as light sources for absorption spectroscopy. One problem in the use of conventional rf linacs has been synchronization of the particle beams and the laser beams. The use of laser

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Table 1
Laser and beam parameters

Item		Value	Unit
Laser power		2	TW
Fwhm pulsewidth		100	fs
	$2.34\sigma_z$	30	μm
Laser wavelength	λ_L	800	nm
Focused spot size	$r_0 = 2\sigma_r$	40	μm
Rayleigh length	z_R	6.35	mm
Normalized vector potential	a_0	0.135	
Pulse frequency		10	Hz
Injection beam energy		10	MeV
Normalized emittance	ϵ_N	π	μm
Beam current		200	A
Energy spread	$\Delta\gamma/\gamma$	0.2	%
Plasma density	n_p	$7.00 \cdot 10^{17}$	cm^{-3}
Plasma wavelength	λ_p	39.8	μm
Relativistic factor of plasma wave	γ_Φ	49.7	
Longitudinal wakefield	eE_z	0.970	GeV m^{-1}
LWA energy gain		$> \pi z_R eE_z > 20.0$	MeV
Rms bunch length		5.4	fs
Bunch frequency		10	Hz
Bunch charge		10	pC

acceleration has another advantage: because the same laser oscillator can be used for both acceleration and spectroscopy, the synchronization problem is readily solved.

In this paper we give a design of a real accelerator based on LWA for such applications. Their requirements other than the bunch length, say, energy, energy width, beam current, etc., are rather modest. The overall specifications are well within a range attainable by a LWA based on the present technology.

The arrangement of this paper is as follows. The Section 2 summarizes the specifications aimed at in this design study. Section 3 gives a sketch of the system which realizes those specifications. Section 4 reviews LWA. It is shown that bunching is possible in the wakefield. However, the acceleration is more efficient if the injection beams have a length as short as the accelerating-focusing phase of the plasma wave. Section 5 gives a design of a prebuncher to realize such injection beams. Section 6 contains discussion, in which necessary research and development are itemized. Some alternative ideas and conclusions are also given.

2. Specifications

Parameters showing the design goal of this paper are summarized in Table 1. We assume use of a commercially-available Ti:sapphire laser as a driver of the laser wakefield and a state-of-art rf photocathode as an injector, the latter of which is the result of KEK-BNL-SHI collaboration and now under test. We expect that this rf photocathode will perform better than the previous model [8]. The plasma density is assumed to mate not with the self-modulated but with the standard LWA.

The charge of a bunch in the Table 1, 10 pC, may sound too small compared with that of a conventional rf linac, typically 1 nC, for pulse radiolysis in the picosecond to nanosecond range. This problem of the bunch charge has already been discussed in [9].

3. System construction

The injection beam to the LWA is imposed by some requirements. First of all, the injected electrons should have a certain amount of energy, otherwise the plasma wave cannot trap the electrons in its wave potential. The threshold energy is a function of wave phase velocity and the wave amplitude, which is shown in Fig. 5 [10]. Output of the rf photocathode can fulfil the requirement of energy. Our parameters of Table 1 give $\gamma_\Phi = 50$ and $eE_z/(m\omega_p c) = 0.121$. The injection energy of $\gamma = 19.6$ is well above the threshold.

If a beam with a current of ~ 1 kA is available, we have two ways to obtain high-charge short bunches, say 10 pC in 10 fs. One is to inject low-energy bunches whose length is less than a quarter of the plasma wavelength in the plasma for the acceleration. The ponderomotive force of a laser excites wakefields in both longitudinal and transverse directions. Accelerating and decelerating phases appear alternately in the longitudinal direction, while focusing and defocusing phases appear alternately in the transverse direction. As shown in Fig. 1, the phases of longitudinal and transverse wakefields shift by $\pi/2$. It is thus ideal that the injection beam to the wakefields is bunched within a quarter of the plasma wavelength, in which the accelerating and focusing phases overlap.

The other way is to inject a much longer bunch than the plasma wavelength, though it is less efficient than the first

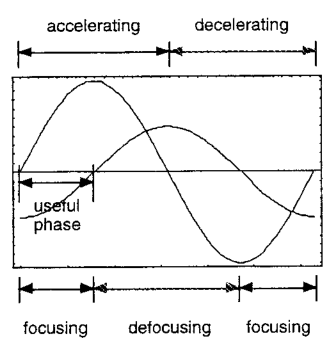

Fig. 1. The longitudinal and transverse wakefields in the plasma wave.

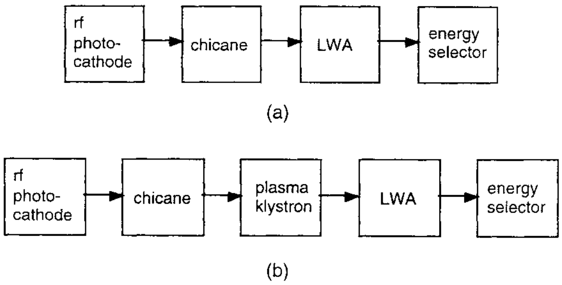

Fig. 2. System construction. In (a), a bunch much longer than the plasma wavelength is introduced into the LWA. The accelerator functions also as a buncher. In (b), a bunch with length $\sim \lambda_p/4$ is introduced to the optimum phase of the plasma wave in the LWA.

way. As described in Section 4, the plasma wave bunches the beam. This is because the phase of the plasma wave can modulate the velocity of the electrons if their relativistic factor is not very high.

If the injected charge exceeds the limit loadable to the plasma wave, we can obtain only a single bunch in both ways.

Fig. 2(a), (b) show the constructions of the respective two ways. As is mentioned later, the first way in Fig. 2(a) requires an injected beam with a current exceeding 80 pC/132 fs = 600 A. The other way in Fig. 2(b) requires an injection bunch with a charge of 10 pC in the length of 30 fs.

The most straightforward method to satisfy the requirement of the first way is to create a 80 pC in 132 fs single bunch directly by an rf photocathode. However, even the shortest bunch ever obtained, 40 pC in 370 fs [8], is far from the values of 80 pC in 132 fs. The technology rf photocathode is developing rapidly, but a more realistic method at the present is to compress the output of the rf photocathode. Recent experiments have demonstrated that the conventional magnetic compression using a chicane can achieve a peak current greater than 1 kA [11]. Though some refinement is necessary, it is not difficult to realize the first way using the technique of magnetic compression.

If we want to realize the second way, we have to further compress this 1 kA chicane output at least by a factor of four. Two candidates are use of an FEL [12], and a plasma klystron [13]. Simulation studies show that the FEL can form a dc beam into bunches with length much shorter than 100 fs [12]. It however requires THz radiation, which makes the design complicated and expensive. Use of the plasma klystron is discussed in Section 5. It produces bunches with a length of $\sim \lambda_p/4$, which are introduced to the optimum phase of the plasma waves in the LWA. The energy width accelerated at the LWA is quite large. An energy selector consisting of a bending magnet is required at the end in both cases.

4. Laser wakefield acceleration

4.1. Acceleration gradient and acceleration length

The ponderomotive force of a high-power short-pulsed laser excites a plasma wave in the LWA. The potential of the wave is used for particle acceleration. The phase velocity of the plasma wave is equal to the group velocity of the laser in the plasma, which is approximately equal to the velocity of light. The plasma wave is consequently adequate to accelerate light particles, such as electrons.

Two LWA methods have been experimentally verified [1–3]: the standard LWA [14,15] and the self-modulated LWA [16,17]. The self-modulated LWA can surely attain a higher acceleration gradient, a shorter bunch length and a larger bunch frequency than those of the standard LWA. It is certainly preferable in our application in these respects. There exist, however, many unknown factors concerning it. Calculations are thus carried out for the standard LWA in this paper.

The standard LWA uses a laser whose rms pulse width (σ_z) is around the value related to the plasma wavelength (λ_p) by the following equation:

$$\lambda_p = \pi \sigma_z. \tag{1}$$

Its maximum acceleration gradient is

$$eE_z = \frac{2\pi^{1/2} m_e c^2 a_0^2}{\exp(1)\sigma_z}, \tag{2}$$

where $a_0 = 0.85 \times 10^{-9} \lambda_L(\mu m) I^{1/2}(W\ cm^{-2})$ is the amplitude of the normalized vector potential of a linear-polarized laser, with λ_L being the laser wavelength and I the laser power density.

It has been said that the acceleration length of the standard LWA is limited by πz_R, where z_R is the Rayleigh length,

$$z_R = \pi r_0^2/\lambda_L, \tag{3}$$

and r_0 is the focused waist size of the laser, which is related to the rms size (σ_r) by the relation $r_0 = 2\sigma_r$.

In Table 1, the plasma density was first selected to mate with the length of the laser pulse. Our recent experimental results suggests that a guiding mechanism of a laser helps to lengthen this length [3]. An energy gain exceeding the value in Table 1 may be obtainable. We, however, base our design here on the equations above, which gives a moderate value.

4.2. Bunching

Fig. 1 suggests that the injection bunch should have the whole length $\lambda_p/4$ which is around 10 μm in our design. Though some ideas are given in Section 4.3, it does not seem easy to realize this bunch length. Fortunately, a plasma wave has the ability to bunch a dc beam, just as a

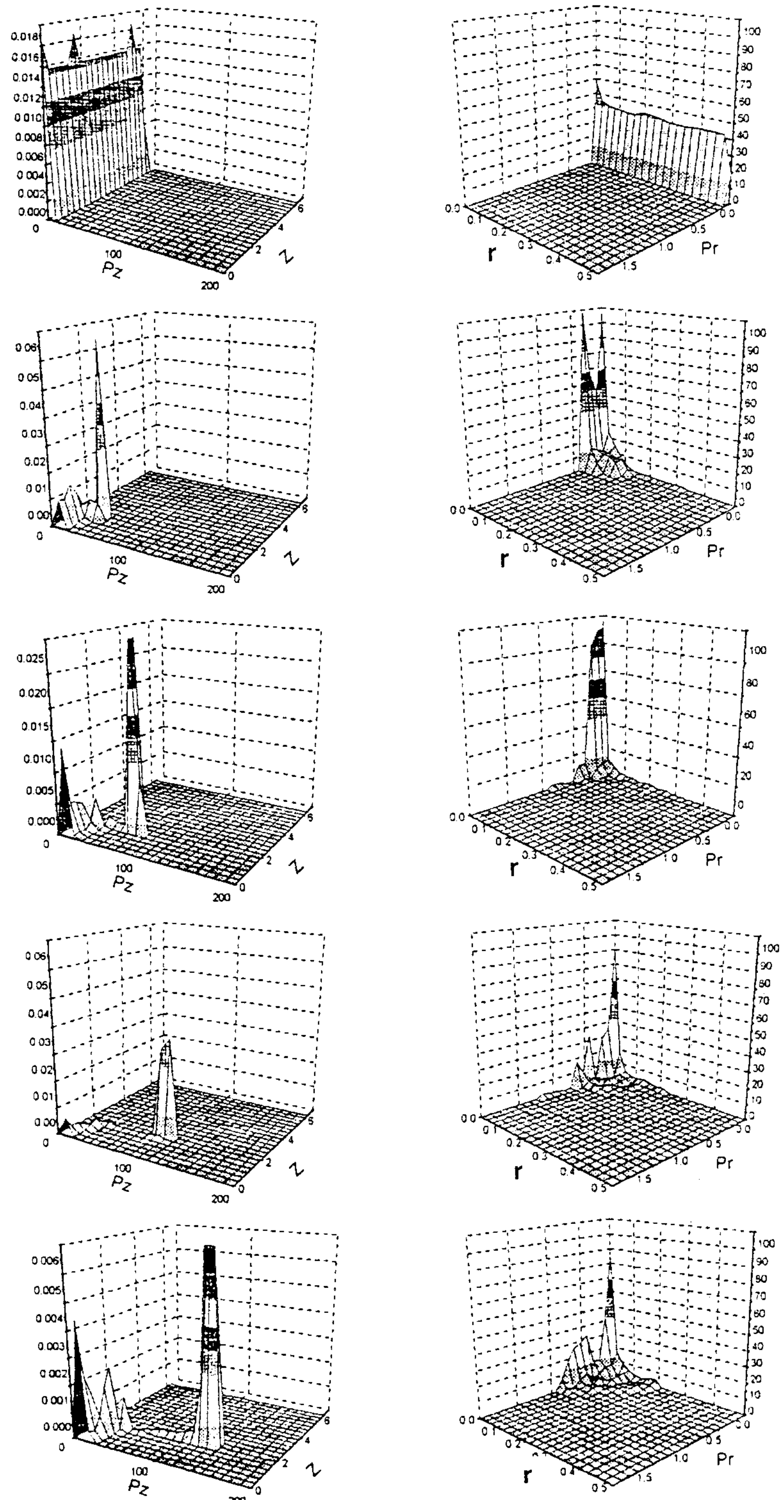

Fig. 3. Bunching and acceleration in the standard LWA without beam loading. Left plots show distributions on $(z,\ p_z/mc)$ space while right plots show those on $(r,\ p_r/mc)$ space. The five pairs of plots are given in the time interval of $\pi z_R/(4c)$.

conventional rf wave does, which is shown in Fig. 3. They are results of a PIC simulation of a standard LWA. Though the parameters used here were not exactly the same as those given in Table 1, those results enable us intuitive understanding of the bunching process.

The number of particles was 10000. All the lengths were normalized to the inverse wavenumber of a plasma wave and the time is given in units of $1/\omega_p$. The longitudinal plasma-wave profile was assumed to be sinusoidal. The relativistic factor of the plasma wave (γ_Φ) was 20. The initial test electrons form a cylinder with uniform distribution, whose energy was given by $\gamma = 2.4$, much smaller than the γ-factor of the plasma waves. The figure shows that a bunch is formed in the early stage of acceleration. The bunching in the linear regime has also been verified by experiments [18].

4.3. Bunching under beam loading

The calculation of Fig. 3 does not take into account the beam loading. It has been known that a particle beam can excite a wakefield in a plasma [19]. Beam loading is just the reverse process of this wakefield excitation [20]. The amount of charge which can be loaded on a given wakefield is derived from the wakefield which a particle with unit charge excites in a plasma. The number of electrons (N_T) acceleratable by the longitudinal wakefield (E_z) is then given by

$$N_T = \epsilon_0 \pi r_0^2 E_z / e, \tag{4}$$

if the bunch length is infinitely short. The resultant beam energy has a 100% width.

As shown in the simple calculation below, it is possible to attain a smaller energy width by sacrificing the number of electrons. Suppose that we have n particles, all of which have the same initial energy and are distributed homogeneously only in the positive side of the longitudinal phase. In other words, the particle distribution is described by a step function with a discontinuity at zero on the z-axis. The kth particle is described by a couple of equations [21],

$$\frac{d\psi_k}{dz} = \frac{2\pi}{\lambda_p}\left(\frac{1}{\beta_p} - \frac{\gamma_k(z)}{\left(\gamma_k^2(z) - 1\right)^{1/2}}\right), \tag{5}$$

$$\frac{d\gamma_k}{dz} = -\frac{e}{m_e c^2}\left(E_z \sin\psi_k(z) - E_{bk}\right). \tag{6}$$

The first term in the rhs parentheses of Eq. (6) is the laser wakefield, while the second term is the wakefield of the particles. A one-dimensional calculation gives the wakefield inside a beam as [22]

$$E_b(z) = \frac{m_e w_p c}{e n_p}\int_{-\infty}^{z} n_b(z)\cos k_p z \, dz, \tag{7}$$

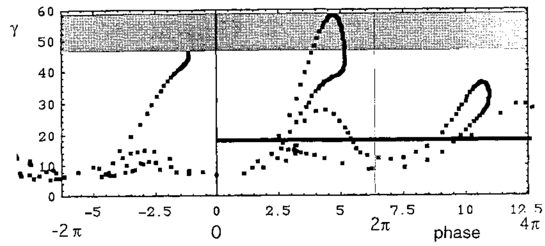

Fig. 4. Longitudinal phase diagrams with beam loading. The value of α was 1.49. The lines parallel to the horizontal axes show the injected beams with a density of 2.5×10^{15} cm^{-3}. By selecting the energy region meshed, we obtain a pure single bunch.

where n_b is the electron density inside a beam. The E_{bk} term is a discrete version of this equation, which is

$$E_{bk}(z) = \frac{m_e w_p c}{e}\frac{n_b}{n_p}\sum_{i=1}^{n}\Delta\psi\cos(\psi_k(z) - \psi_i(z)),$$
$$\psi_k(z) > \psi_i(z). \tag{8}$$

It should be noted that source of the wakefield caused by a particle beam is the space charge force. We can regard the term $E_{bk}(z)$ as an expression of the space charge effect.

Fig. 4 shows longitudinal phase diagrams in phase between -2π and 4π under beam loading, in which each kth particle is depicted by a dot. The values of α (capture parameter) defined by

$$\alpha = \frac{m_e w_p c}{e E_z}\left(\frac{1-\beta_0}{1+\beta_0}\right)^{1/2}, \tag{9}$$

was 1.79, corresponding to $\gamma_0 = 18$. The line parallel to the horizontal axes shows the injected beams, which does not exist in the negative-phase regions. The electron beam density is assumed to be 2.5×10^{15} cm^{-3}, which is smaller than the plasma density in the order of two magnitudes.

The figure shows that some decelerated particles spill out from the first plasma-wave bucket, and are accelerated

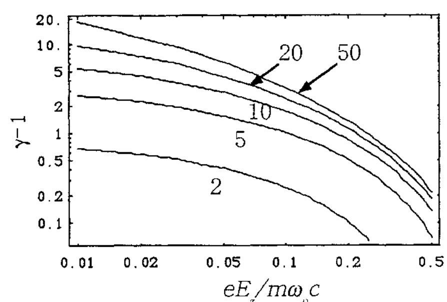

Fig. 5. Threshold energy of electrons to be trapped by the potential of the plasma wave. Horizontal axis eE_z/mw_pc is the normalized plasma wakefield. Parameter is $\gamma_\Phi = \lambda_p/\lambda_L$, the relativistic factor of the plasma wave.

and bunched in the preceding bucket, where they are free from the wakefields of the preceding beams. The number of spill-out particles decreases as the initial beam energy increases, or α decreases. In the case of this figure, we can obtain a pure single bunch by selecting those particles with $\gamma > 48$, because only the particles in the main wave, $0 < \psi < 2\pi$, at the middle of the figure, have such energies. This energy region is covered by meshes in the figure.

The particles in this meshed region form a single bunch with a 25% energy width. The total bunch length is $\sim \pi/5 = 8$ μm or 26 fs. If we approximate the longitudinal distribution as being triangular, the rms bunch length is $26/(24)^{1/2} \sim 5.4$ fs. This simulation shows that a quarter of the injected beam is compressed into this bunch. The electron-beam volume before compression in one plasma wavelength is $\sim \pi(40\ \mu\text{m})^2 \times 40\ \mu\text{m} = 200 \times 10^{-9}\ \text{cm}^3$. The injected beam current should be 80 $\text{pC}/\lambda_p/c = 600$ kA. The charge of this single bunch then becomes around 20 pC. The resultant energy width is fairly large in this design. Certainly the applications given in Section 1 do not require fine energy resolution, but we should note that other applications may require it.

5. Plasma klystron

As was shown in Fig. 2, we have two ways to obtain high-charge bunches from the LWA. One way in Fig. 2(a) is to inject a bunch much longer than the plasma wavelength. The other way in Fig. 2(b) is to inject bunches whose length is less than a quarter of the plasma wavelength in the plasma for the acceleration. They should have 10 pC in the length of 30 fs. We discuss the use of a plasma klystron to produce such bunches.

The plasma klystron in the original form is shown in Fig. 6(a). It consists of a short plasma section with length δ followed by a free space with length z_0. A weak plasma wave is excited by the same laser that is used for acceleration. The plasma should have the same density as the main plasma for acceleration. A long beam is energy-modulated at the prebuncher plasma. The slippage Δz caused by this energy modulation with amplitude $\Delta\gamma$ is given by

$$\frac{\Delta z}{z_0} = \frac{1}{\gamma^2} \frac{\Delta\gamma}{\gamma}, \tag{10}$$

where

$$\Delta\gamma = 2\delta \frac{eE_z}{m_e c^2}, \tag{11}$$

and E_z is the amplitude of laser wakefield. We require

$$\Delta z = \lambda_p/2. \tag{12}$$

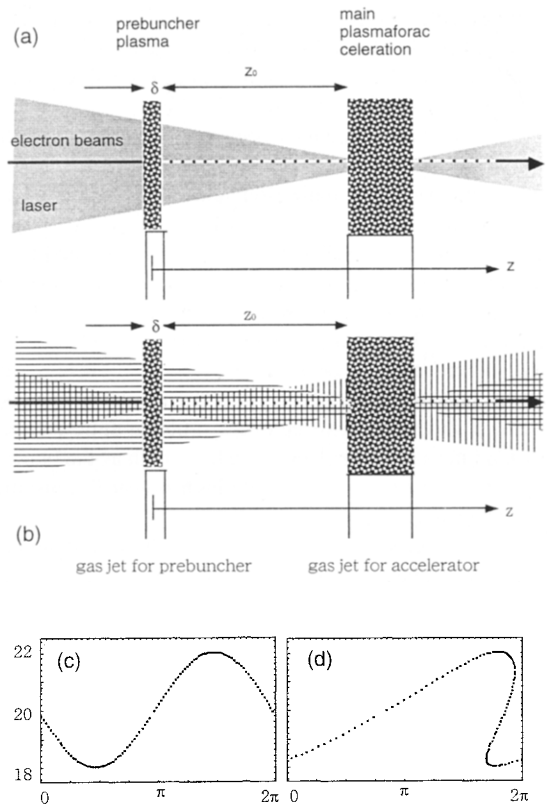

Fig. 6. Plasma klystron. (a) original scheme using one laser beam. (b) scheme using two laser beams. Waveforms give longitudinal phase-space diagrams at the exit of a prebuncher plasma (c) and at the entrance of acceleration plasma (d).

This relation determines the thickness of the prebuncher plasma as

$$\delta = \frac{\gamma^3}{4} \frac{\lambda_p}{z_0} \frac{m_e c^2}{eE_z}. \tag{13}$$

The above calculation does not take account of the space charge effect. The bunch lengthening due to this effect can be roughly estimated by the following equation [13]:

$$\Delta L = \frac{2Qcz_0^2}{I_A \gamma^4 RL}, \tag{14}$$

where $I_A = 17$ kA is the Alfven current, and the bunch profile was approximated by a cylinder with radius R and length L.

The following should be considered in the design of the plasma klystron:

- The energy modulation should be small, preferably within ±10% of the injection energy.
- The strength and radius of the wakefield in the buncher should stand the beam loading given in Eq. (4).

● The bunch lengthening due to space charge effect given in Eq. (14) should be less than 1 μm/pC.

● The laser intensity should exceed the threshold of tunneling ionization, typically 10^{15} W cm^{-2}.

The fourth item is added because we assume here that the plasma is created by the same laser for acceleration due to multi-photon ionization and tunneling ionization processes [1,2,23,24].

The number of controllable parameters is limited in the original plasma klystron shown in Fig. 6(a), because this scheme is for formation of longer bunches in beatwave acceleration. The design of Fig. 6(b) was thus adopted here, where two focal points, each for bunching and acceleration, are used. Although the figure shows two laser beams, it should be possible to construct an optical system in which the same laser beam functions both as the buncher and the accelerator, because little laser power is consumed in the buncher.

Two waveforms in Fig. 6(c–d) show longitudinal phase-space diagrams at the exit of a prebuncher plasma and at the entrance of acceleration plasma, respectively. The energy of the injection beam and the density of the prebuncher plasma used for the calculation were consistent with those given in Table 1. The bunching laser has the same focal spot size and same power as the accelerating laser, although its length (δ) is limited to 1.05 mm by the width of the gas jet. This figure shows that more than 70% of the electrons are bunched within phase span of $\pi/8$. The bunch lengthening due to the space charge effect is 8.83 μm in 10 pC bunch with $R = 40$ μm, $L = 10$ μm, $\gamma = 20$ and $z_0 = 40$ mm.

6. Discussion

The hitherto description reveals that a femtosecond linac with moderate energy based on the LWA is technically feasible. There, however, remain quite a few items to be studied and developed, which include the following.

● Basic LWA experiments scanning the laser radius, the laser energy, etc. Focusing has been very tight in the hitherto experiments, because the acceleration gradient had the first priority to be aimed at. The present design loosed the focus so that the energy distribution of the accelerated beams is flat, but the effect of focusing should be examined experimentally.

● Basic gas jet experiments. Our recent experiments suggests that, in a gas-filled chamber, the self-channeling effect lengthens the acceleration length and that a kind of self-modulation different from those hitherto observed enhances the acceleration gradient. We can expect a much higher energy gain than that given in Table 1, if these positive results are reproducible is gas-jet plasmas, which has to be used in a plasma klystron.

● Basic plasma klystron experiments. There has been no experiments concerning it yet.

● A simulation code which takes account of an electron beam in addition to a plasma and a laser. It aims at studying beam loading, the bunching process and the two-dimensional dynamics, including the plasma lens effect.

● Study of electron beam transfer from the acceleration chamber to the sample without degrading the beam quality. Because the bunch length is so short, effect of coherent radiation generation should be taken into account, especially at the energy selector in Fig. 2.

The present design is based on the standard LWA. The beat wave acceleration could give a similar design [25]. The self-modulated LWA has higher acceleration gradient to bear higher beam loading. Its acceleration gradient could be close to, or even higher than, the cold-wave breaking limit, $E_z = m_e \omega_p c/e$. A plasma density of 4×10^{19} cm^{-2} satisfies the two conditions for a self-modulated [16,17], if the laser described in Table 1 is used. First, the laser pulse length (30 μm) must be longer than the plasma wavelength (5.28 μm), and second, the laser power (2 TW) must exceed the critical power for relativistic optical guiding (0.47 TW). According to PIC simulations, the number of plasma waves excited by the self-modulated LWA is quite small, which reduces the number of bunches. Assuming a laser spot size of 10 μm and using Eq. (4), we find that the acceleratable charge is on the order of nC in the self-modulated LWA. The bunch length could be much shorter than the plasma wavelength (17 fs). However, it enhances technical difficulties. First, the injection-beam current should be on the order of 10 kA to form a single bunch. Second, the electron beam size should be reduced so as to mate with the plasma wavelength.

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